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On The Total Vertex Irregularity Strength of Series Parallel Graph $sp(m, r, 3)$

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A B S T R A C T

This paper addresses the problem of determining the total vertex irregularity strength of the series-parallel graph family $sp(m, r, 3)$ for $m \geq 4$ and $r \geq 2$. The total vertex irregularity strength $tvs(G)$ of a graph G is defined as the smallest integer k such that there exists a total k-labelling $f: V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ where the vertex weights $w(v) = f(v) + \sum_{x \in N(v)} f(vx)$ are distinct for each vertex. The graph family $sp(m, r, 3)$ is generated through repeated series and parallel compositions, with parameters m, r, and a fixed structural parameter 3. To solve this problem, we construct an explicit total labelling that ensures distinct vertex weights, providing an upper bound for $tvs(sp(m, r, 3))$. Additionally, we perform a structural analysis of the graph, which yields a matching lower bound. The results demonstrate that the total vertex irregularity strength of $sp(m, r, 3)$ is given by $tvs(sp(m, r, 3)) = \lceil (3mr + 2) / 2 \rceil$. This work contributes a new insight into the characterization of the total vertex irregularity strength for this specific class of graphs, providing both upper and lower bounds for $sp(m, r, 3)$.

Penelitian ini membahas permasalahan dalam menentukan nilai total ketakteraturan titik pada keluarga graf seri-paralel $sp(m, r, 3)$ untuk $m \geq 4$ dan $r \geq 2$. Nilai total ketakteraturan titik $tvs(G)$ untuk suatu graf G didefinisikan sebagai nilai minimum k sehingga terdapat pelabelan total $f: V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ dengan bobot titik $w(v) = f(v) + \sum_{x \in N(v)} f(vx)$ yang berbeda untuk setiap titik. Keluarga graf $sp(m, r, 3)$ dibangun melalui komposisi seri dan paralel secara berulang, dengan parameter m, r, dan parameter struktur tetap 3. Untuk menyelesaikan masalah ini, kami mengonstruksi pelabelan total eksplisit yang memastikan bobot titik saling berbeda, sehingga menghasilkan batas atas untuk $tvs(sp(m, r, 3))$. Selain itu, kami melakukan analisis struktur graf untuk memperoleh batas bawah yang sesuai. Hasil penelitian ini menunjukkan bahwa total vertex irregularity strength untuk $sp(m, r, 3)$ diberikan oleh $tvs(sp(m, r, 3)) = \lceil (3mr + 2) / 2 \rceil$. Penelitian ini memberikan kontribusi berupa wawasan baru dalam karakterisasi nilai total ketakteraturan titik untuk kelas graf ini, dengan menyediakan batas atas dan batas bawah untuk $sp(m, r, 3)$.

1. Introduction

Graphs, which consist of multiple vertices and edges connecting them, can be used to model a wide range of real-world situations. It is important to note that a graph represents whether a pair of vertices is connected by an edge. The definition of graphs arises from the mathematical abstraction of such conditions [1]. In graph labelling, a labelling function is defined as a mapping with an explicit domain and codomain. Specifically, for a graph $G = (V, E)$, a (vertex) labelling is a function $f: V \to \mathbb{N}$, an edge labelling is a function $f: E \to \mathbb{N}$, and a total labelling is a function $f: V \cup E \to \mathbb{N}$ (or equivalently $f: V \cup E \to \{1,2, \ldots, k\}$ for some integer k) [2]. Graph labeling has been used in many applications like communication network addressing, software testing, information security, technology and sports tournament scheduling, and coding theory problems including the design of good radar location codes, missile guidance codes, and convolution codes, secret sharing methods, and models for constraint programming across finite domains, labeled graphs are helpful models [3].

Graph G is characterized as a set match, composed with the notation $G = (V, E)$, where V is a non-empty set of points (vertices or nodes), and E is the set of lines (edges or arcs) that join two vertices. Discrete items and the relationships between them are represented by graphs [4]. Graph labelling is the process of assigning labels, which are often represented by integers, to the edges and/or vertices of a graph in the mathematical field of graph theory [3].

The idea of k-labelling, which is irregular labelling on a graph G defined as a mapping of a set of edge e of G to an integer $\{1,2,\dots,k\}$ such that each vertex v has a distinct weight, was first described in[5]. The weighted of the vertex v, denoted by $w(v)$, is the sum of the labels v and the labels of edges linked with v. The weighted edge e, denoted as $w(e)$, is the sum of labels e and labels for all vertices connected to e. Several studies related to vertices include graceful vertex labeling on graphs (5,8)[6], graceful vertex labeling on graphs (5,7)[7] and graceful vertex labeling on graphs (7,8)[8].

The concept of examining irregular total k-labelling was presented in [9]. A vertex irregular total k-labelling of G is defined as follows: for every two distinct vertices x and y of G, there is $w(x) \neq w(y)$. Similarly, an edge irregular total k-labelling of graph G is defined as follows: for every two distinct edges e and f of graph G, there is $w(e) \neq w(f)$. The graph G's total edge irregularity strength, represented by $tes(G)$, is the lowest k for which the graph has an edge irregular total k-labelling. In a similar vein, we define $tvs(G)$, the total vertex irregularity strength of G, as the lowest k for which a vertex irregular total k-labelling of G exists.

The study of irregular total labelling in a graph continues to grow. In a study [10], the irregularity strength of the Diamond graph Br_n was examined, where, for $n \geq 3$, $tes(Br_4) = \left\lceil \frac{5n-3}{3} \right\rceil$ and $tvs(Br_4) = \left\lceil \frac{n+1}{3} \right\rceil$. The total edge irregularity strength of the Kite graph (n,t) was then presented in[11], which demonstrated that for $n \geq 3$ and $t \geq 1$, showed $tes(n,t) = \left\lceil \frac{n+t+2}{3} \right\rceil$. The total edge irregularity strength of centralized uniform theta graphs, $\theta^*(n;m;p)$, was published later in [2] and yielded the following conclusion for $n \geq 3$, $m \geq 1$, and $p \geq 3$: $tes(\theta * (n;m;p)) = \left\lceil \frac{(n(m+1)p+2)}{3} \right\rceil$. Then, according to study in [12], the total irregularity strength of the comb product of a two-cycle graph C_m and C_n is $tvs(C_m \rhd_o C_n) = \left\lceil \frac{m(n-1)+2}{3} \right\rceil$ for $m \geq 3$ and $n \geq 3$, whereas the total irregularity value of a two-star graph S_m and S_n is $tvs(S_m \rhd_o S_n) = \left\lceil \frac{n(m+1)+1}{2} \right\rceil$ for $m \geq 2$ and $n \geq 2$. The total irregularity edge strength of the m-copy of the path graph P_n is $tes(mP_n) = \left\lceil \frac{(n-1)m+2}{3} \right\rceil$, for $m \geq 2$ and $n \geq 6$, were presented in [13].

One of the results of the total irregularity strength of the tadpole chain graph $T_r(4,1)$ was $tvs(T_r(4,1)) = \left\lceil \frac{4r+2}{5} \right\rceil$ for $r \geq 3$. This was demonstrated in[14]. The total edge irregularity strength for the ladder graph SC_n, double ladder graph DSC_n with $tes(DSC_n) = \left\lceil \frac{2n^2+3n+1}{3} \right\rceil$, mirror ladder graph MSC_n, and double ladder graph DSC_n, with $tes(MSC_n) = \left\lceil \frac{n(2n+5)+2}{3} \right\rceil$, was determined by study [15]. In his study [16], Hinding looks at a hexagon cluster network $HC(n)$'s total vertex irregularity strength and finds the values for $tvs(HC(n)) = \left(\frac{3n^2+1}{2}\right) n \geq 2$.

Some attention was also paid to the total edge irregularity strength of parallel series graphs. A series parallel graph $sp(m,r,l)$ has an overall edge irregularity value of [17], which can be found by writing $tes(sp(m,r,l)) = \left\lceil \frac{lm(r+1)+2}{3} \right\rceil$ for $r \geq 1$. This is one of the writings by Winarsih. The total irregularity vertex on a series parallel graph has been determined by Marzuki, et. al. [18] and Riskawati [19] in their respective publications. For $m \geq 4$ on the graph $sp(m,1,3)$, the value is $tvs(sp(m,1,3)) = \left\lceil \frac{3m+2}{3} \right\rceil$, and for $m \geq 3$ and $r \geq 3$, the value on the graph $sp(m,r,2)$ is $tvs(sp(m,r,2)) = \left\lceil \frac{2mr+2}{3} \right\rceil$. The overall vertex irregularity strength of a parallel series graph was found in those studies to be $sp(m,r,3)$ for $m \geq 4$ and $r \geq 2$.

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2. Methods

In this paper, we consider the finite undirected graph G without loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex x is the number of edges that have x as an endpoint, and the set of neighbours of x is denoted by $N(x)$. If the domain of the labelling function $f(x)$ is the vertex set or the edge set, the labelling is called, respectively, vertex labelling or edge labelling. If the domain is $V(G) \cup E(G)$, then we call the labelling a total labelling.

Regard to a total $k-$labelling of G, that is, $f: V(G) \cup E(G) \longrightarrow \{1, 2, 3, \cdots, k\}$. The associated vertex weight of a vertex $x \in V(G)$ under a total $k-$labelling f is defined as $w(x) = f(x) + \sum_{v \epsilon N(x)} f(xv)$. A total $k-$labelling f is defined to be a vertex irregular total $k-$labelling of G if for every two different vertices x and y of G, $wt(x) \neq wt(y)$. The minimum positive integer k for which G has a vertex irregular total $k-$labelling is called the total vertex irregularity strength of G, denoted by $tvs(G)$.

Assume that the graph $G = (V, E)$ consists of a set of edges connecting a pair of vertices (E) to a non-empty set of vertices (V). If a graph is made up of a series composition of three uniform theta graphs, with m representing each theta graph's longitude and r denoting the number of degree 2 vertices that cross each longitude, it is referred to as a parallel series graph $sp(m, r, 3)$. Illustration of graph $sp(m, r, 3)$ is given in Figure 1 below.

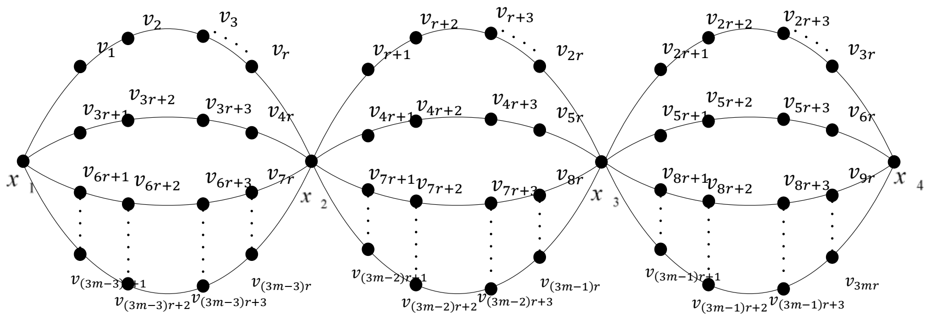

Figure 1. Graph illustration $sp(m, r, 3)$

The graph $sp(m, r, 3)$ has a defined set of vertices, $V = \{v_i: i = 1,2,3, \dots, 3mr\} \cup \{x_i: i = 1,2,3,4\}$. A collection of vertices from the graph are divided into the following groups to make the process of creating edge labelling and determining vertex weight simpler:

a. set of vertices with v_i ; $i = 1, 3r + 1, 6r + 1, 9r + 1, \dots, (3m - 3)r + 1$
b. set of vertices with v_i ; $i = r, 4r, 7r, 10r, \dots, (3m - 2)r$
c. set of vertices with v_i ; $i = r + 1, 4r + 17r + 1, 10r + 1, \dots, (3m - 2)r + 1$
d. set of vertices with v_i ; $i = 2r, 5r, 8r, 11r, \dots, (3m - 1)r$
e. set of vertices with v_i ; $i = 2r + 1, 5r + 1, 8r + 1, 11r + 1, \dots, (3m - 1)r + 1$
f. set of vertices with v_i ; $i = 3r, 6r, 9r, 12r, \dots, 3mr$
g. set of vertices with v_i ; $i = 3jr + 2, 3jr + 3, 3jr + 4, \dots, 3jr + (r + 1)$ and $j = 0,1,2,3, \dots, m - 1$
h. set of vertices with v_i ; $i = 3jr + (r + 2), 3jr + (r + 3), 3jr + (r + 4), \dots, 3jr + (2r - 1)$ and $j = 0,1,2,3, \dots, m - 1$
i. set of vertices with v_i ; $i = 3jr + (2r + 2), 3jr + (2r + 3), 3jr + (2r + 4), \dots, 3jr + (3r - 1)$ and $j = 0,1,2,3, \dots, m - 1$

The set of edges E of the graph is defined, where $sp(m, r, 3)$

$E = \{x_1 v_i: i = 1, 3r + 1, 6r + 1, 9r + 1, \dots, (3m - 3)r + 1\} \cup$
$\{x_2 v_i: i = r + 1, 4r + 17r + 1, 10r + 1, \dots, (3m - 2)r + 1\} \cup$

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$\{x_3v_i: i = 2r + 1, 5r + 1, 8r + 1, 11r + 1, \ldots, (3m - 1)r + 1\} \cup$
$\{x_2v_i: i = r, 4r, 7r, 10r, \ldots, (3m - 2)r\} \cup$
$\{x_3v_i: i = 2r, 5r, 8r, 11r, \ldots, (3m - 1)r\} \cup$
$\{x_4v_i: i = 3r, 6r, 9r, 12r, \ldots, 3mr\} \cup$
$\{v_iv_{i+1}: i = 1,2,3, \ldots, 3mr, i \neq r, 2r, 3r, \ldots, 3mr\}$.

We obtained the lower bound of graph $sp(m, r, 3)$ by analysing the structure of the graph, then the largest minimum label of k or the upper bound k is analysed by labelling the vertices and edges of the graph. By obtaining the biggest lower bound and the smallest upper bound, the total vertex irregularity of the series parallel graph is determined.

3. Result and Discussion

The result of this research is about the total vertex irregularity value of the graph $sp(m, r, 3)$ for $m \geq 4$ and $r \geq 2$ given in the following theorem.

Theorem 1 Total vertex irregularity of the graph $sp(m, r, 3)$ for $m \geq 4$and $r \geq 2$ is

$$tvs(sp(m, r, 3)) = \left\lceil \frac{3mr + 2}{3} \right\rceil$$

Proof. We will prove $tvs(sp(m, r, 3)) \geq \left\lceil \frac{3mr+2}{3} \right\rceil$. Note that the degree of the smallest vertex of the graph $sp(m, r, 3)$ is 2 and the number of vertices with the smallest degree, which is degree 2 on the graph $sp(m, r, 3)$, is $3mr$. To obtain optimal labelling, the weight of each vertex with degree 2 are labelled as $3,4,5, \ldots, 3mr + 2$. Since the vertex weight is the sum of labels of 1 vertex and 2 edges which associated with that vertex, the largest label is more or equal to $\left\lceil \frac{3mr+2}{3} \right\rceil$. The ceiling function is used because in irregular total labelling of vertices, it is only allowed to label the graph with an integer. To guarantee this, the lower bound is rounded up. Then it is evident that $tvs(sp(m, r, 3)) \geq \left\lceil \frac{3mr+2}{3} \right\rceil$.

Next, it will be proved that $tvs(sp(m, r, 3)) \leq \left\lceil \frac{3mr+2}{3} \right\rceil$ by showing the vertex irregular total *k*-labelling of the graph $sp(m, r, 3)$ for m natural numbers and $m \geq 4$, that is

1) The vertex labelling of the graphs $sp(m, r, 3)$ for $m \geq 4$ and $r \geq 2$

a. $$\lambda(v_i) = \left\lceil \frac{i}{3} \right\rceil = \begin{cases} \frac{i+2}{3} & ; \text{for } i \equiv 1 \ (mod\ 3) \\ \frac{i+1}{3} & ; \text{for } i \equiv 2 \ (mod\ 3) \\ \frac{i}{3} & ; \text{for } i \equiv 0 \ (mod\ 3) \end{cases}$$

b. $$\lambda(x_1) = \begin{cases} 6r - 1 & ; \text{if } m = 4 \\ 5r - 2 & ; \text{if } m = 5 \\ 3r - 3 & ; \text{if } m = 6 \\ 1 & ; \text{if } m \geq 7 \end{cases}$$

c. $\lambda(x_2) = 1$

d. $$\lambda(x_3) = \begin{cases} 1 & ; \text{for } r \equiv 1 \ (mod\ 3) \\ 2 & ; \text{for } r \equiv 2 \ (mod\ 3) \\ 1 & ; \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

e. $$\lambda(x_4) = \begin{cases} 2r + 2 & ; \text{if } m = 4 \\ 1 & ; \text{if } m \geq 5 \end{cases}$$

2) The edge labels of the graphs $sp(m, r, 3)$ for $m \geq 4$ and $r \geq 2$

a. For $i = 1, 3r + 1, 6r + 1, 9r + 1, \ldots, (3m - 3)r + 1$

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$$\lambda(x_1v_i)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+1}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

b. For $i=r,4r,7r,10r,\dots,(3m-2)r$

$$\lambda(v_ix_2)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+4}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

c. For $i=r+1,4r+1,7r+1,\dots,(3m-2)r+1$

$$\lambda(x_2v_i)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+1}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

d. For $i=2r,5r,8r,11r,\dots,(3m-1)r$

$$\lambda(v_ix_3)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+4}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

e. For $i=2r+1,5r+1,8r+1,11r+1,\dots,(3m-1)r+1$

$$\lambda(x_3v_i)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+1}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

f. For $i=3r,6r,9r,12r,\dots,3mr$

$$\lambda(x_4v_i)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+4}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

g. For $i=r,4r,7r,\dots,(3m-2)r;$
$2r,5r,8r,\dots,(3m-1)r;$
$3r,6r,9r,\dots,3mr$

$$\lambda(v_{i-1}v_i)=\begin{cases}\frac{i+2}{3} & ;\text{for } i\equiv 1\ (mod\ 3)\\ \frac{i+1}{3} & ;\text{for } i\equiv 2\ (mod\ 3)\\ \frac{i+3}{3} & ;\text{for } i\equiv 0\ (mod\ 3)\end{cases}$$

h. For $i=1,3r+1,6r+1,\dots,(3m-3)r+1;$
$r+1,4r+1,7r+1,\dots,(3m-2)r+1;$
$2r+1,5r+1,8r+1,\dots,(3m-1)r+1$

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$$\lambda(v_iv_{i+1}) = \begin{cases} \frac{i+2}{3} & ;\text{for } i \equiv 1\ (mod\ 3) \\ \frac{i+4}{3} & ;\text{for } i \equiv 2\ (mod\ 3) \\ \frac{i+3}{3} & ;\text{for } i \equiv 0\ (mod\ 3) \end{cases}$$

i. For $i = 3jr + 2, 3jr + 3, 3jr + 4, \dots, 3jr + (r-1);$
$3jr + (r+2), 3jr + (r+3), 3jr + (r+4), \dots, 3jr + (2r-1);$
$3jr + (2r+2), 3jr + (2r+3), 3jr + (2r+4), \dots, 3jr + (3r-1);$

$$\lambda(v_{i-1}v_i) = \begin{cases} \frac{i+2}{3} & ;\text{for } i \equiv 1\ (mod\ 3) \\ \frac{i+1}{3} & ;\text{for } i \equiv 2\ (mod\ 3) \\ \frac{i+3}{3} & ;\text{for } i \equiv 0\ (mod\ 3) \end{cases}$$

$$\lambda(v_iv_{i+1}) = \begin{cases} \frac{i+2}{3} & ;\text{for } i \equiv 1\ (mod\ 3) \\ \frac{i+4}{3} & ;\text{for } i \equiv 2\ (mod\ 3) \\ \frac{i+3}{3} & ;\text{for } i \equiv 0\ (mod\ 3) \end{cases}$$

According to the given labelling, the vertex weights v_i for graph $sp(m,r,3)$ where $m \geq 4$ and $r \geq 2$ are denoted by $w(v_i)$ which equals $i+2$. The weight of the vertex v_i ranges from 3 to $3mr + 2$ as consecutive integers, proving that each vertex weight v_i is unique in the graphs $sp(m,r,3)$ for $m \geq 4$ and $r \geq 2$. Next will be calculated the vertex weight x_i, with $i = 1,2,3,4$, from the graph $sp(m,r,3)$, and it will be proven that each vertex weight $v_i$and each vertex weight $x_i$in the series–parallel graph $sp(m,r,3)$are distinct for $m \geq 4$ and $r \geq 2$.

1. For $m = 4$ and $r \geq 2$

$$wt(x_1) = 12r + 3$$

$$wt(x_2) = \begin{cases} \frac{44r+19}{3} & \text{for } r \equiv 1\ (mod\ 3) \\ \frac{44r+35}{3} & \text{for } r \equiv 2\ (mod\ 3) \\ \frac{44r+27}{3} & \text{for } r \equiv 0\ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \frac{52r+35}{3} & \text{for } r \equiv 1\ (mod\ 3) \\ \frac{52r+22}{3} & \text{for } r \equiv 2\ (mod\ 3) \\ \frac{52r+27}{3} & \text{for } r \equiv 0\ (mod\ 3) \end{cases}$$

$$wt(x_4) = 12r + 6$$

For $m = 4$, the vertex weights v_i for $i = 1,2,3, \dots, 3mr$ are consecutive integers from $3,4,5, \dots, 12r + 2$. The following will show that

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

For $r \equiv 1(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$12r + 2 < 12r + 3 < 12r + 6 < \frac{44r+19}{3} < \frac{52r+35}{3}.$$

For $r \equiv 2(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

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holds because

$$12r + 2 < 12r + 3 < 12r + 6 < \frac{44r + 35}{3} < \frac{52r + 22}{3}.$$

For $r \equiv 0(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$12r + 2 < 12r + 3 < 12r + 6 < \frac{44r + 27}{3} < \frac{52r + 27}{3}.$$

These results show that every vertex weight in the graph $sp(m, r, 3)$for $m = 4$ is distinct.

2. For $m = 5$ and $r \geq 2$

$$wt(x_1) = 15r + 3$$

$$wt(x_2) = \begin{cases} \frac{70r + 23}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{70r + 43}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{70r + 33}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \frac{80r + 43}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{80r + 26}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{80r + 33}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_4) = 15r + 6$$

For $m = 5$, the vertex weights v_i for $i = 1,2,3, \dots, 3mr$ are consecutive integers from $3,4,5, \dots, 15r + 2$. The following will show that

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

For $r \equiv 1(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$15r + 2 < 15r + 3 < 15r + 6 < \frac{70r + 23}{3} < \frac{80r + 43}{3}.$$

For $r \equiv 2(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$15r + 2 < 15r + 3 < 15r + 6 < \frac{70r + 43}{3} < \frac{80r + 26}{3}.$$

For $r \equiv 0(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$15r + 2 < 15r + 3 < 15r + 6 < \frac{70r + 33}{3} < \frac{80r + 33}{3}.$$

These results show that every vertex weight in the graph $sp(m, r, 3)$ for $m = 5$ is distinct.

3. For $m = 6$ and $r \geq 2$

$$wt(x_1) = 18r + 3$$

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$$wt(x_2) = \begin{cases} \frac{102r+27}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{102r+51}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{102r+39}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \frac{114r+27}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{114r+51}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{114r+39}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_4) = 21r + 7$$

For $m = 6$, the vertex weights v_i for $i = 1,2,3, \ldots, 3mr$ are consecutive integers from $3,4,5, \ldots, 18r + 2$. The following will show that

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

For $r \equiv 1(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$18r + 2 < 18r + 3 < 21r + 7 < \frac{102r+27}{3} < \frac{114r+51}{3}.$$

For $r \equiv 2(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$18r + 2 < 18r + 3 < 21r + 7 < \frac{102r+51}{3} < \frac{114r+30}{3}.$$

For $r \equiv 0(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$18r + 2 < 18r + 3 < 21r + 7 < \frac{102r+39}{3} < \frac{114r+39}{3}.$$

These results show that every vertex weight in the graph $sp(m, r, 3)$ for $m = 6$ is distinct.

4. For $m = 7$ and $r \geq 2$

$$wt(x_1) = 21r + 8$$

$$wt(x_2) = \begin{cases} \frac{140r+31}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{140r+59}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{140r}{3} + 15 & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \frac{154r+59}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{154r+34}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{154r}{3} + 15 & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_4) = 28r + 8$$

For $m = 7$, the vertex weights v_i for $i = 1,2,3, \ldots, 3mr$ are consecutive integers from $3,4,5, \ldots, 21r + 2$. The following will show that

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

For $r \equiv 1(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$21r + 2 < 21r + 8 < 28r + 8 < \frac{140r + 31}{3} < \frac{154r + 59}{3}.$$

For $r \equiv 2(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$21r + 2 < 21r + 8 < 28r + 8 < \frac{140r + 59}{3} < \frac{154r + 34}{3}.$$

For $r \equiv 0(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$21r + 2 < 21r + 8 < 28r + 8 < \frac{140r}{3} + 15 < \frac{154r}{3} + 15.$$

These results show that every vertex weight in the graph $sp(m, r, 3)$ for $m = 7$ is distinct.

5. For $m = 8$ and $r \geq 2$

$$wt(x_1) = 28r + 9$$

$$wt(x_2) = \begin{cases} \dfrac{184r + 35}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \dfrac{184r + 67}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \dfrac{184r}{3} + 17 & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \dfrac{200r + 67}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \dfrac{200r + 38}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \dfrac{200r}{3} + 17 & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_4) = 36r + 9$$

For $m = 8$, the vertex weights v_i for $i = 1,2,3, \dots, 3mr$are consecutive integers from $3,4,5, \dots, 24r + 2$. The following will show that

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

For $r \equiv 1(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$24r + 2 < 28r + 9 < 36r + 9 < \frac{184r + 35}{3} < \frac{200r + 67}{3}.$$

For $r \equiv 2(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$24r + 2 < 28r + 9 < 36r + 9 < \frac{184r + 67}{3} < \frac{200r + 38}{3}.$$

For $r \equiv 0(\text{mod}3)$ with $r \geq 2$, the inequality

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3)$$

holds because

$$24r + 2 < 28r + 9 < 36r + 9 < \frac{184r}{3} + 17 < \frac{200r}{3} + 17.$$

These results show that every vertex weight in the graph $sp(m, r, 3)$ for $m = 8$ is distinct.

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6. For $m \geq 9$ and $r \geq 2$

$$wt(x_1) = \frac{m^2 - mr + 2m + 2}{2}$$

$$wt(x_2) = \begin{cases} \frac{3m^2r - mr + 4m + 3}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{3m^2r - mr + 8m + 3}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{3m^2r - mr + 6m + 3}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_3) = \begin{cases} \frac{3m^2r + mr + 8m + 3}{3} & \text{for } r \equiv 1 \ (mod\ 3) \\ \frac{3m^2r + mr + 4m + 6}{3} & \text{for } r \equiv 2 \ (mod\ 3) \\ \frac{3m^2r + mr + 6m + 3}{3} & \text{for } r \equiv 0 \ (mod\ 3) \end{cases}$$

$$wt(x_4) = \frac{m^2 - mr + 2m + 2}{2}$$

For $m \geq 9$, the vertex weights v_i for $i = 1,2,3,\ldots,3mr$ are consecutive integers from

$$3,4,5,\ldots,27r + 2.$$

The following will be shown:

$$wt(v_{3mr}) < wt(x_1) < wt(x_4) < wt(x_2) < wt(x_3).$$

Case 1: $r \equiv 1(\text{mod}3)$ with $r \geq 2$

We will prove that

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2} < \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 4m + 3}{3} < \frac{3m^2r + mr + 8m + 3}{3}.$$

The inequalities

$$\frac{m^2r - mr + 2m + 2}{2} < \frac{m^2r + mr + 2m + 2}{2}, \frac{3m^2r - mr + 4m + 3}{3} < \frac{3m^2r + mr + 8m + 3}{3}$$

are trivially true because the right-hand sides are larger than the left-hand sides.

Thus, the nontrivial steps requiring proof are:

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2}, \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 4m + 3}{3},$$

which will be established using mathematical induction.

Induction Proof 1

To prove:

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2} \text{ for } m \geq 9.$$

Let

$$p(m): 3mr + 2 < \frac{m^2r - mr + 2m + 2}{2}.$$

Base Step

Because $m \geq 9$, we check $m = 9$:

$$3mr + 2 = 27r + 2 < \frac{72r + 20}{2} = \frac{9^2r - 9r + 2 \cdot 9 + 2}{2} = \frac{m^2r - mr + 2m + 2}{2}.$$

Thus, $p(9)$ is true.

Induction Step

Assume $p(k)$holds:

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$$3kr + 2 < \frac{k^2r - kr + 2k + 2}{2}.$$

We must prove:

$$3kr + 3r + 2 < \frac{k^2r + kr + 2k + 4}{2}.$$

Adding $3r$ to both sides of the induction hypothesis yields:

$$3kr + 3r + 2 < \frac{k^2r - kr + 6r + 2k + 2}{2}.$$

Since

$$\frac{2kr - 6r + 2}{2} \geq 0,$$

we have

$$\frac{k^2r - kr + 6r + 2k + 2}{2} < \frac{k^2r + kr + 2k + 4}{2}.$$

Thus, the inequality for $k + 1$ holds and the induction is complete.

Induction Proof 2

To prove:

$$\frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 4m + 3}{3} \text{for } m \geq 9.$$

Let

$$p(m): \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 4m + 3}{3}.$$

Base Step

For $m = 9$:

$$\frac{90r + 20}{2} < \frac{234r + 39}{3}.$$

So $p(9)$ holds.

Induction Step

Assume

$$\frac{k^2r + kr + 2k + 2}{2} < \frac{3k^2r - kr + 4k + 3}{3}.$$

We need to prove:

$$\frac{k^2r + 3kr + 2r + 2k + 4}{2} < \frac{3k^2r + 5kr + 2r + 4k + 7}{3}.$$

Adding $kr + r + 1$ to both sides of the assumption gives the desired result, completing the induction.

Case 2: $r \equiv 2 (\mathrm{mod} 3), r \geq 2$

The inequalities to be shown are:

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2} < \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 8m + 3}{3} < \frac{3m^2r + mr + 4m + 6}{3}.$$

The inequalities

$$\frac{m^2r - mr + 2m + 2}{2} < \frac{m^2r + mr + 2m + 2}{2}, \frac{3m^2r - mr + 8m + 3}{3} < \frac{3m^2r + mr + 4m + 6}{3}$$

are clearly true, since the right-hand side is larger than the left-hand side. However, the inequalities

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2}, \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 8m + 3}{3}$$

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require proof using mathematical induction.

Induction Proof 1

To prove:

$$3mr + 2 < \frac{m^2r - mr + 2m + 2}{2} \text{ for } m \geq 9.$$

Let

$$p(m): 3mr + 2 < \frac{m^2r - mr + 2m + 2}{2}.$$

Base Case

Since $m \geq 9$, we check $m = 9$:

$$3mr + 2 = 27r + 2 < \frac{72r + 20}{2} = \frac{9^2r - 9r + 2 \cdot 9 + 2}{2} = \frac{m^2r - mr + 2m + 2}{2}.$$

Thus, $p(9)$ holds.

Induction Step

Assume $p(k)$ holds:

$$3kr + 2 < \frac{k^2r - kr + 2k + 2}{2}.$$

We must prove:

$$3kr + 3r + 2 < \frac{k^2r + kr + 2k + 4}{2}.$$

Adding $3r$ to both sides of the induction hypothesis gives:

$$3kr + 3r + 2 < \frac{k^2r - kr + 6r + 2k + 2}{2}.$$

Since

$$\frac{2kr - 6r + 2}{2} \geq 0,$$

we obtain:

$$\frac{k^2r - kr + 6r + 2k + 2}{2} < \frac{k^2r + kr + 2k + 4}{2}.$$

Thus, the inequality for $k + 1$holds, and the induction is complete.

Induction Proof 2

To prove:

$$\frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 8m + 3}{3} \text{ for } m \geq 9.$$

Let

$$p(m): \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 8m + 3}{3}.$$

Base Case

For $m = 9$:

$$\frac{m^2r + mr + 2m + 2}{2} = \frac{90r + 20}{2} < \frac{234r + 75}{3} = \frac{3 \cdot 9^2r - 9r + 8 \cdot 9 + 3}{3} = \frac{3m^2r - mr + 8m + 3}{3}.$$

Thus, $p(9)$ is true.

Induction Step

Assume $p(k)$ holds:

$$\frac{k^2r + kr + 2k + 2}{2} < \frac{3k^2r - kr + 8k + 3}{3}.$$

We must prove:

$$\frac{k^2r + 3kr + 2r + 2k + 4}{2} < \frac{3k^2r + 5kr + 2r + 8k + 11}{3}.$$

Adding $kr + r + 1$ to both sides of the induction hypothesis yields:

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$$\frac{k^2r+3kr+2r+2k+4}{2}<\frac{3k^2r+2kr+3r+4k+6}{3}.$$

Since

$$\frac{3kr-r+5}{3}\geq 0,$$

we obtain:

$$\frac{3k^2r+2kr+3r+4k+6}{3}<\frac{3k^2r+5kr+2r+8k+11}{3}.$$

Thus, the inequality for $k+1$ holds, completing the induction.

Case 3: $r\equiv 0(\mathrm{mod}3), r\geq 2$

The inequalities to be shown are:

$$3mr+2<\frac{m^2r-mr+2m+2}{2}<\frac{m^2r+mr+2m+2}{2}<\frac{3m^2r-mr+8m+3}{3}<\frac{3m^2r+mr+4m+6}{3}.$$

The inequalities

$$\frac{m^2r-mr+2m+2}{2}<\frac{m^2r+mr+2m+2}{2},\frac{3m^2r-mr+8m+3}{3}<\frac{3m^2r+mr+4m+6}{3}$$

are clearly true, since the right-hand side is greater than the left-hand side. The inequalities that require proof are

$$3mr+2<\frac{m^2r-mr+2m+2}{2},\frac{m^2r+mr+2m+2}{2}<\frac{3m^2r-mr+8m+3}{3},$$

and these will be proven using mathematical induction.

Induction Proof 1

To prove:

$$3mr+2<\frac{m^2r-mr+2m+2}{2}\text{ for } m\geq 9.$$

Let

$$p(m): 3mr+2<\frac{m^2r-mr+2m+2}{2}.$$

Base Case

Since $m\geq 9$, check $m=9$:

$$3mr+2=27r+2<\frac{72r+20}{2}=\frac{9^2r-9r+2\cdot 9+2}{2}=\frac{m^2r-mr+2m+2}{2}.$$

Thus, $p(9)$ holds.

Induction Step

Assume $p(k)$ holds:

$$3kr+2<\frac{k^2r-kr+2k+2}{2}.$$

We must prove:

$$3kr+3r+2<\frac{k^2r+kr+2k+4}{2}.$$

Adding $3r$ to both sides yields:

$$3kr+3r+2<\frac{k^2r-kr+6r+2k+2}{2}.$$

Because

$$\frac{2kr-6r+2}{2}\geq 0,$$

we obtain:

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$$\frac{k^2r - kr + 6r + 2k + 2}{2} < \frac{k^2r + kr + 2k + 4}{2}.$$

Thus, $p(k+1)$ holds, completing the induction.

Induction Proof 2

To prove:

$$\frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 6m + 3}{3} \text{for } m \geq 9.$$

Let

$$p(m): \frac{m^2r + mr + 2m + 2}{2} < \frac{3m^2r - mr + 6m + 3}{3}.$$

Base Case

For $m = 9$:

$$\frac{m^2r + mr + 2m + 2}{2} = \frac{90r + 20}{2} < \frac{234r + 75}{3} = \frac{3 \cdot 9^2r - 9r + 8 \cdot 9 + 3}{3} = \frac{3m^2r - mr + 8m + 3}{3}.$$

Thus, $p(9)$ is true.

Induction Step

Assume $p(k)$holds:

$$\frac{k^2r + kr + 2k + 2}{2} < \frac{3k^2r - kr + 6k + 3}{3}.$$

We must prove:

$$\frac{k^2r + 3kr + 2r + 2k + 4}{2} < \frac{3k^2r + 5kr + 2r + 6k + 9}{3}.$$

Adding $kr + r + 1$to both sides of the induction hypothesis gives:

$$\frac{k^2r + 3kr + 2r + 2k + 4}{2} < \frac{3k^2r + 2kr + 3r + 6k + 6}{3}.$$

Since

$$\frac{3kr - r + 3}{3} \geq 0,$$

it follows that:

$$\frac{3k^2r + 2kr + 3r + 6k + 6}{3} < \frac{3k^2r + 5kr + 2r + 6k + 9}{3}.$$

Thus, the inequality for $k + 1$ holds.

The calculation of the vertex weight shows that $w(v_{3mr}) < w(x_1) < w(x_4) < w(x_2) < w(x_3)$, indicating that each vertex in graph $sp(m,r,3)$ has a unique weight. Therefore, it can be deduced that in the vertex irregular total labelling of the graph $sp(m,r,3)$, each vertex has a distinct weight and $tvs(sp(m,r,3)) \leq \lceil (3mr+2)/3 \rceil$. From the above explanation, we can conclude that $tvs(sp(m,r,3)) \geq \lceil (3mr+2)/3 \rceil$ and $tvs(sp(m,r,3)) \leq \lceil (3mr+2)/3 \rceil$, thus proving that $tvs(sp(m,r,3)) = \lceil (3mr+2)/3 \rceil$.

As an illustration of the above theorem, an example is given for labelling the irregular totals of vertices for the graphs $sp(m,r,3)$ for $m = 13$ and $r = 2$.

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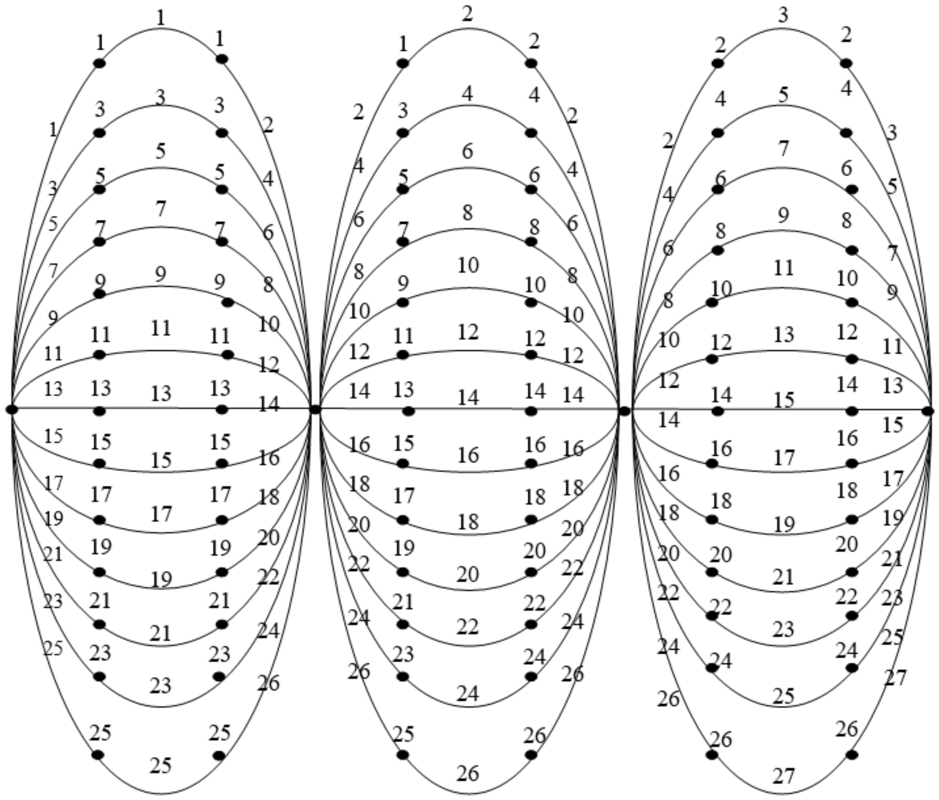

Figure 2. Graph illustration $sp(13, 2, 3)$

4. Conclusion

Based on the description above, it can be concluded that there is an irregular total $\left\lceil\frac{3mr+2}{3}\right\rceil$-labelling of vertices on a parallel series graph $sp(m,r,3)$ for $m \geq 4$ and $r \geq 2$ and an irregular total $\left\lceil\frac{4mr+2}{3}\right\rceil$-labelling of vertices on a parallel series graph $sp(m,r,4)$ for $m \geq 5$ and $r \geq 1$. Hence, it is proven that $tvs(sp(m,r,3)) \geq \left\lceil\frac{3mr+2}{3}\right\rceil$. As the lower bound has been obtained, and the upper bound of $tvs(sp(m,r,3))$, so that $tvs(sp(m,r,3)) \leq \left\lceil\frac{3mr+2}{3}\right\rceil$.

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